What do mesons have to do with nuclear structure?

Daniel S. Koltun

Department of Physics and Astronomy, University of Rochester

Abstract

The theory of nuclear structure (binding, low energy spectra, transitions, etc.) depends on nucleon-nucleon (NN) interactions. The meson theory of NN interactions has predictive power for NN scattering, and partial success when applied to the theory of nuclear structure. Is it possible to test this theoretical picture – by direct experimental interaction with mesons in nuclei? Some experimental searches for the 'pion excess' in nuclei have ambiguous results. How sensitive are such experiments to the mesonic aspects of nuclear structure? These questions are addressed in this talk. More details and references are given in [1].

1 Meson exchange interactions in nuclei

The one pion exchange (OPE) potential for momentum transfer $\boldsymbol{q}$ is given by

$$V_\pi(q) = -\frac{f^2}{{m_\pi}^2}\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{(q^2+{m_\pi}^2)}\tau_1\cdot\tau_2. \tag{1}$$

The analogous (spin) potential for exchange of one rho meson can be written

$$V_\rho(q) = -\frac{{f_\rho}^2}{{m_\rho}^2}\frac{(\boldsymbol{\sigma}_1\times\boldsymbol{q})\cdot(\boldsymbol{\sigma}_2\times\boldsymbol{q})}{(q^2+{m_\rho}^2)}\tau_1\cdot\tau_2. \tag{2}$$

These are both important contributors to the NN tensor force and to nuclear binding. For a nucleus A, the meson contributions are

$$< V_{\text{meson}} >_A = \int \frac{d^3q}{(2\pi)^3} < V_{\text{meson}}(q) >_A, \tag{3}$$

where, for the pion

$$< V_\pi(q) >_A = -\frac{f^2}{2m_\pi^2}\frac{S_L^{(2)}(q)}{(q^2+m_\pi^2)}, \tag{4}$$

with the longitudinal spin correlation function given by the expectation value in the nuclear ground state

$$S_L^{(2)}(q) = \frac{1}{3Aq^2}\sum_{k\neq j}^{A} < (\boldsymbol{\sigma}_k \cdot \boldsymbol{q})(\boldsymbol{\sigma}_j \cdot \boldsymbol{q})\tau_k \cdot \tau_j \exp[i\boldsymbol{q}\cdot(\boldsymbol{r}_k - \boldsymbol{r}_j)] >_A . \tag{5}$$

Similarly, the rho contribution depends on the transverse spin correlation function

$$S_T^{(2)}(q) = \frac{1}{6Aq^2}\sum_{k\neq j}^{A} < (\boldsymbol{\sigma}_k \times \boldsymbol{q})\cdot(\boldsymbol{\sigma}_j \times \boldsymbol{q})\tau_k \cdot \tau_j \exp[i\boldsymbol{q}\cdot(\boldsymbol{r}_k - \boldsymbol{r}_j)] >_A . \tag{6}$$

For example, calculations with the Argonne-Urbana NN potentials and full correlations [2] give

$$< V_\pi >_A /A = -30.7 \text{ MeV} \tag{7}$$

for ^{16}O. This is a major part of the nuclear binding.

2 Inelastic excitation of nuclei

Nuclear structure enters inelastic scattering through response functions, defined as follows:

$$R_\alpha^a(q,\omega) = \sum_f |< f|\Gamma_\alpha^a(\boldsymbol{q})|i >|^2 \delta(\omega - E_f + E_i), \tag{8}$$

where i and f denote initial and final nuclear states. In the present context we discuss two: the longitudinal $R_L^a(q,\omega)$ and transverse $R_T^a(q,\omega)$, which correspond to exchange (between the projectile and the nuclear target) of 0^- and 1^- quanta, pion-like, or rho-like. The $\Gamma_\alpha^a(\boldsymbol{q})$ are given by the following single-nucleon operators:

$$\Gamma_L^a(\boldsymbol{q}) = \sum_{k=1}^{A}(\boldsymbol{\sigma}_k \cdot \boldsymbol{q})\tau_k^a \quad \exp(i\boldsymbol{q}\cdot\boldsymbol{r}_k), \tag{9}$$

$$\Gamma_T^a(\boldsymbol{q}) = \sqrt{\frac{1}{2}}\sum_{k=1}^{A}(\boldsymbol{\sigma}_k \times \boldsymbol{q})\tau_k^a \quad \exp(i\boldsymbol{q}\cdot\boldsymbol{r}_k), \tag{10}$$

a) Spin transfer to a nuclear target by the $(\vec{p},\vec{n})$ reaction can be analyzed, under some assumptions about the reaction, to give two response functions, $R^+_\alpha(q,\omega)$, with $a=+$. For a $T=0$ target, these are related to the full isovector response functions by $R_\alpha(q,\omega)\equiv\sum_a R^a_\alpha(q,\omega)=3R^+_\alpha(q,\omega)$.

b) Inelastic scattering on virtual pions (or other mesons) can be related to the imaginary part of the forward amplitude

$$\mathcal{I}m\ \mathcal{F}(0)=-\frac{1}{4\pi}\int\frac{d^4q}{(2\pi)^4}P(q)\mathcal{I}m\ \mathcal{M}(p,q), \tag{11}$$

using p,q for the 4-momenta of the projectile and the meson. Here $P(q)$ is an invariant distribution function for the virtual meson, and $\mathcal{M}$ is the invariant scattering amplitude on the meson. Sullivan gave the result for pions [3], which can be written for pseudovector coupling to nucleons with form factor $F(t)$:

$$P(q)=\pi\frac{f^2}{m_\pi^2}F^2(t)\frac{R_L(q,\omega)}{(t+m_\pi^2)^2}, \tag{12}$$

where $t=q^2-\omega^2$. Thus the pion contribution is related to the response R_L.

3 Pion distribution functions and pion excess

Pion distributions are connected to nuclear structure through the response functions. The pion probability distribution in the nuclear target (A) is given by:

$$n_A(q)=<\sum_a a^+_a(\boldsymbol{q})a_a(\boldsymbol{q})>_A\ . \tag{13}$$

More of interest for nuclear structure is the *excess* distribution, defined by the difference $\delta n_A(q)=n_A(q)-An_N(q)$, where $n_N(q)$ is for a single nucleon. This quantity is not directly measured in experiment, but it is connected to the function $\delta R_L=R_L^{(A)}-AR_L^{(N)}$. In a static approximation, and assuming pseudovector coupling of pions to nucleons, one finds

$$\delta n_A(q)=\frac{f^2F^2(q^2)}{2\varepsilon_q^3m_\pi^2}\int_0^\infty d\omega\delta R_L(q,\omega)=\frac{3Aq^2f^2F^2(q)}{2\varepsilon_q^3m^2}S_L^{(2)}(q) \tag{14}$$

with pion energy $\varepsilon_q=\sqrt{q^2+m_\pi^2}$. The last term follows from the sum rules which connect the response functions to the correlation functions:

$$\begin{aligned}S_\alpha(q)&\equiv\frac{1}{3Aq^2}\int_0^\infty d\omega R_\alpha(q,\omega)\\&=\frac{1}{3Aq^2}<\sum_a\Gamma^{a\dagger}_\alpha(\boldsymbol{q})\Gamma^a_\alpha(\boldsymbol{q})>_A=1+S^{(2)}_\alpha(q).\end{aligned} \tag{15}$$

To the same approximation, $\delta n_A(q) = -< V_\pi(q) >_A / \varepsilon_q$. The integrated excess per nucleon is $\delta n_A/A \simeq 0.03$ in the theory which gives (7) [2, 4].

A quantity which is used in the analysis of DIS experiments is the momentum distribution function for pions in the target. This may be defined, using the scaling variable y,

$$p_A(y) = y \int \frac{d^4q}{(2\pi)^4} P(q) \delta(y - \frac{(q_z - \omega)}{M}), \tag{16}$$

with M the nucleon mass and q_z the longitudinal component of $\boldsymbol{q}$.

4 What is known about response functions?

The response of a noninteracting Fermi gas (FG) of nucleons is given by an inverted parabola in ω (for $q \geq 2k_F$), whose peak value (when normed to unity) is given by

$$\max \frac{R^{FG}}{3Aq^2} = \frac{3M}{4k_F q}, \quad \text{at } \omega = \frac{q^2}{2M}, \tag{17}$$

and a range $\Delta\omega = 2qk_F/M$. For the mean-field shell model (SM) the response is qualitatively similar to that for the FG, governed by the particle-hole spectrum. The peak position is usually shifted above the FG value of $\omega = q^2/2M$.

For interacting nuclear systems in general, calculation of the response functions is difficult. However, the two sum-rule functions $S_\alpha(q), W_\alpha(q)$ can be obtained as ground state expectation values. S_α is given by (15); the energy-weighted sum rule is given by

$$W_\alpha(q) = \frac{1}{3Aq^2} \int_0^\infty d\omega \omega R_\alpha(q,\omega) = \frac{1}{6Aq^2} < \sum_a [\Gamma_\alpha^{a\dagger}(\boldsymbol{q}), [H, \Gamma_\alpha^a(\boldsymbol{q})] >_A . \tag{18}$$

These have been calculated for various nuclear ground states by the Argonne-Urbana group, using realistic NN interactions [4].

Direct calculation of the response has been done for a few cases:

In RPA [5, 6], with π- and ρ-exchange, with strong attraction at short range (Landau $g' \simeq 0.7$) one gets an enhanced FG or SM peak for R_L, suppressed peak for R_T, but no broadening, since the same p-h spectrum as in the SM dominates the energy dependence. This reflects the sum rule behavior, $S_L \geq S^{SM} \geq S_T$, for the implied tensor correlations.

In the Correlated Basis Function method [7], NN correlations are included in the ground and p-h excited states. The response peaks are shifted and broadened by the correlations, but the spectrum is still based on the p-h assumption. The strengths S_L and S_T are similar to the Argonne-Urbana results.

Both above theories simulate the mixing of higher configurations (2p-2h) by broadening the p-h energies. This adds a high-energy tail to the ω dependence.

A direct calculation of a scalar response function for ^{4}He has been done by an integral transform method [8], but only for central NN interactions, which is not suitable for the spin-dependent R_L, R_T. However, these calculations give large strength for ω well above the SM peak - a high-energy tail. Additional evidence of strength at high energy is given by [4], who calculate the Euclidean (Laplace) transforms of both R_L and R_T, and for $A > 4$.

5 Sum rules and models of response functions

Here I describe a model for the functional dependence of the response functions, using information directly from the two sum rules, which are more easily calculated than the response itself (See [1]). The basic idea is to separate the response functions into two components, the first of which characterizes the noninteracting part, and which gives the SM peak. A second distribution includes the effects of correlations, and provides strength for ω above the SM peak. We write

$$R_\alpha(q,\omega) = R_\alpha^{SM}(q,\omega) + \Delta R_\alpha(q,\omega), \tag{19}$$

$$S_\alpha(q) = S^{SM}(q) + \Delta S_\alpha(q), \tag{20}$$

$$W_\alpha(q) = W^{SM}(q) + \Delta W_\alpha(q). \tag{21}$$

One can calculate S_α and W_α for interacting nuclei [4], and S^{SM} for shell model nuclei. W^{SM} is more model dependent, but can be estimated. Then ΔS_α fixes the integrated strength, and ΔW_α the centroid, of the correlation response ΔR_α. One needs to assume the functional forms. Values of these quantities are given for ^{16}O in Tables I and II of [1].

For ΔR_L we consider two models: in the first we take the distribution to be constant, e.g., for $0 < \omega < 2\omega_L(q)$:

$$\frac{\Delta R_L(q,\omega)}{3Aq^2} = \frac{\Delta S_L(q)}{2\omega_L(q)}\Theta(2\omega_L(q) - \omega). \tag{22}$$

This model has a symmetric distribution in ω about the centroid $\omega_L(q) = \Delta W_L(q)/\Delta S_L(q)$. The second distribution is not symmetric:

$$\frac{\Delta R_L(q,\omega)}{3Aq^2} = \frac{\Delta S_L(q)}{\beta^2}\omega \exp(-\omega/\beta). \tag{23}$$

with $2\beta = \omega_L(q)$. This form has its maximum value at $\omega = \beta = \omega_L(q)/2$.

For R_T we need a simple model with a sign change at some ω_0; we take

$$\frac{\Delta R_T(q,\omega)}{3Aq^2} = \frac{\Delta W_T(q)}{\omega_0^2}[-\Theta(\omega_0-\omega)+\Theta(\omega-\omega_0)\Theta(2\omega_0-\omega)]. \qquad (24)$$

6 Nuclear correlations and $(\vec{p},\vec{n})$ data

Using the model forms just described and the calculated values for ΔS_L and ΔW_L, we find the following estimates for the values of the ΔR_α (normalized for comparison to experimental values). We find that $\omega_L = 293$ MeV for the range of interest, 200 MeV/$c \leq q \leq 400$ MeV/c. For the constant model (22) we find

$$\Delta R_L/3Aq^2 = 2.6\times 10^{-4}\mathrm{MeV}^{-1} \qquad (25)$$

for all ω, while for the exponential model (23) we obtain an upper bound

$$\Delta R_L/3Aq^2 \leq 3.8\times 10^{-4}\mathrm{MeV}^{-1}. \qquad (26)$$

For the model of ΔR_T in (24), assuming that $\omega_0 = \omega_L$, the value for $\omega \leq \omega_L$ is given by

$$\Delta R_T/3Aq^2 = -4.5\times 10^{-4}\mathrm{MeV}^{-1}. \qquad (27)$$

These estimates may be compared to the data from the $(\vec{p},\vec{n})$ experiments in the range 240 MeV/$c \leq q \leq 380$ MeV/c, which show peak values (see [9], Fig. 2)

$$\max R_\alpha/3Aq^2 \simeq 1.2 - 1.7\times 10^{-2}\mathrm{MeV}^{-1}, \qquad (28)$$

dropping to $\simeq 0.5\times 10^{-2}\mathrm{MeV}^{-1}$ towards the ends of the range of energy losses. The quoted values are dependent on the scattering model used to extract the response functions: see [6] for a different analysis of the same data. But the magnitudes are reasonable; they are of the same order as the value (17) of the quasifree peak for a Fermi gas (nuclear matter) at these momenta.

Our estimates of the correlation contributions to the response functions are then of the order of a few percent at the peak values, and less than 10% over the whole range of excitation energies, $\omega \leq 150$ MeV. These contributions are *smaller* than the estimated uncertainties in the data quoted, including counting ($\leq 10\%$), experimental systematic ($6\%-8\%$), and model uncertainties in extraction ($20\%, 10\%$).

So these experiments are not presently accurate enough to measure the effects expected from nuclear correlations, which is unfortunate. What is curious about the $(\vec{p},\vec{n})$ data is that the values of R_L (which are connected to the pion excess) are not hard to understand, but those for R_T seem to be enhanced. This is not explained by any conventional nuclear mechanism, and has suggested, e.g., evidence for rho enhancement in nuclei [10].

7 Pion contributions to DIS

The estimates of the distributions ΔR_L also have consequences for the analysis of deep inelastic scattering (DIS) or related processes on nuclear targets. The quantity of interest is the pion momentum distribution for the target, $p_A(y)$, defined in (16). This quantity is integrated over y, weighted by the pion structure function, to give the pion contribution to the DIS. Integrating (16) over the three-momentum gives

$$p_A(y) = \frac{f^2}{16\pi^2 m_\pi^2} y \int_{(My)^2}^{\infty} dq^2 \int_0^{\omega_m} d\omega \frac{F^2(t) R_L(q,\omega)}{(t+m^2)^2}, \tag{29}$$

where $y = (q_z - \omega)/M$. The upper limit of the ω-integral is given by

$$\omega_m \equiv q - yM \geq q_z - yM. \tag{30}$$

The main point is the strong effect of the upper limit ω_m on the contribution of the nuclear response function to the ω-integral. To illustrate the effect, making a static approximation as in (14), consider

$$J(q,y) = \frac{1}{3Aq^2} \int_0^{\omega_m} d\omega R_L(q,\omega). \tag{31}$$

Clearly, $J(q,y) \leq S_L(q)$, the suppression increasing with y. As an example, consider $q = 400$ MeV/c, for which the value of $S_L(q) = 1.11$ is a maximum, as is the pion excess, also. One finds that the values of $J(q,y)$ are reduced below 1.0 for $y \simeq 0.3$.

The consequence is that the effect of excess pions which are associated with NN correlations may be sufficiently reduced by kinematic constraints to be inaccessible in DIS or dimuon experiments [11].

8 Conclusions and discussion

So, what do we learn? We have seen that there is a theoretical connection based on the meson theory of NN interactions, between nuclear binding and certain inelastic scattering experiments which have been regarded as sensitive to the pion excess in nuclei. The connection depends on the spin-isospin correlations in nuclei and the response functions R_L and R_T which characterize the scattering.

In the conventional theory of nuclear structure for which the ground states and excitation spectra are dominated by two-nucleon correlations, the major changes in the response functions (compared to the quasifree response) are

expected to be small in magnitude, and spread over a large range of ω. As a consequence, we estimate that these changes in R_L and R_T are well within the uncertainties for the data from the recent $(\vec{p}, \vec{n})$ experiments, for $\omega \leq 150$ MeV.

In the analysis of DIS and dimuon production experiments, part of the high-ω tails for R_L are cut off by kinematical constraints. This can reduce – even eliminate – the effect of excess pions from the structure functions.

So, there is no apparent conflict between the conventional theory of the pion excess and present experimental data, but there is no positive confirmation, either. The response functions are still the key to testing the conventional picture of nuclear correlations and mesons in nuclei, but the requirements for experimental and theoretical accuracy are not easily met.

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